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Accurate measurements of sphere drag at low Reynolds numbers

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An accurate measurement of low-Reynolds-number sphere drag has been made. Some of the inaccuracies of previous measurements are revealed. Comparison with the theoretical studies shows that the Oseen formula is as accurate as any in predicting sphere drag below a Reynolds number of 0·4.

1. Introduction

Considering the large number of studies already performed on the present subject (see Perry 1950 for an exhaustive list), it might be thought the height of pedantry to perform yet another. However, it is felt that several good reasons can be advanced for the execution of the present experiment.

During the course of an experiment to measure the drag experienced by a sphere as it moved slowly through a rotating, viscous fluid (Maxworthy 1965), it was found that the available low-Reynolds-number-drag data, without rotation, was too inaccurate to be acceptable in reducing the new data. From the data plotted in Goldstein (1937, p. 16) or Perry (1950, p. 1018), it would appear that the presently available data is sufficient to accurately answer any reasonable question. However, when the original data is plotted 'correctly'; that is, the drag is non-dimensionalized with respect to the Stokes drag, startling inaccuracies appear. It is in fact impossible to be sure of the drag to better than $\pm 20\,\%$, even if one is very generous with data that scatters between the Stokes drag and values above the Oseen drag. The difficulties faced by previous investigators seemed to be mainly due to an inability to accurately compensate for wall effects. Other problems of experimental determination of fluid properties, etc., also increase scatter beyond reasonable bounds.

The present investigation was undertaken to remedy this situation and check the range of usefulness of the applicable asymptotic theories in predicting the drag at Reynolds numbers of order unity. These theories of 'matched expansions' or 'inner and outer expansions' have been developed to a remarkable degree recently and have been found useful in a variety of circumstances; the existence of at least one experimental check might well be of use in evaluating the range of usefulness under other circumstances.

The 'falling-sphere' technique was used to measure the drag. When a sphere falls at an equilibrium velocity, its drag equals the net gravitational force on the sphere $D = \frac{4}{3}\pi a^3(\rho_s - \rho_f)g$, where a is the sphere radius, ρ_s the density, ρ_f the fluid density, and g the acceleration due to gravity. This must be non-dimensionalized with respect to Stokes drag $D_s = 6\pi\rho_f \nu a U$, where ν is the fluid kinematic viscosity, and U the equilibrium sphere velocity, so that

$$D/D_s = \frac{2}{9}\frac{a^2(\rho_s-\rho_f)g}{\rho_f \nu U}$$

and is a function of Ua/ν only. For the values of sphere and container diameter finally chosen, wall effects make negligible contributions to the drag (i.e. less than 0·2 %).

2. Apparatus and experimental technique

2.1. *Measurement of sphere fall velocity*

Accurate temperature control is essential to the success of the experiment, and the apparatus shown in figure 1 was fabricated to ensure it. The whole apparatus is constructed of Lucite. The main cylinder (A) is filled with a glycerine–water mixture with nominal 10 cS. viscosity at room temperature. An outer, insulated cylinder (B) is the constant-temperature water bath which keeps the fluid temperature constant to within ± 0·01 °C. Ports provide access for an accurate, calibrated thermometer, the sphere retrieval system, and the sphere dispenser. Eighteen sapphire spheres of 1/64 in. nominal diameter were used in all tests. They were dropped and their rates of fall timed through three 40 cm test distances to ± 1/20 sec using a stop watch. Experiments were performed at nine temperatures, and fifty-four individually timed intervals were taken at each; these were averaged and one value of sphere velocity obtained at each temperature.

2.2. *Measurement of sphere and fluid properties*

Four properties must be measured as accurately as the fall velocity, they are: a, ρ_s, ρ_f and ν.

The sphere diameter was measured to $\pm 1 \times 10^{-5}$ in. using an optical interferometric technique. Measurement of several diameters on some of the spheres showed each of them to be round to this accuracy. The sphere density was found by weighing the eighteen spheres on a microbalance to $\pm 1 \times 10^{-4}$ g, and dividing by the total volume of all the spheres. ρ_f was measured using a conventional pychnometer to $\pm 1 \times 10^{-4}$ g/c.c. at several convenient temperatures. An Ostwald viscometer, calibrated using standard N.B.S. viscometer oils, was used to measure fluid viscosity at several temperatures within the range of the experiments. The accuracy of this determination is $\pm \frac{1}{2}$ %. The overall accuracy of the total experiment is better than ± 2 % under the worst conditions, which occur at the higher Reynolds numbers.

3. Results and conclusions

Figure 2 summarizes the results as concisely as possible. Several interesting conclusions can be drawn. Despite the large scatter of all of the previous experiments, the overall average curve is very close to the present determination at Reynolds numbers greater than 1·5. Only at Reynolds numbers below this do serious differences arise. The approach to $R \rightarrow 0$ is the most interesting in this regard. Previous results suggest that the approach is via the Stokes drag, whereas the present results show a definite approach via the Oseen value. Thus in reality Stokes drag is only applicable at $R = 0$ itself and not at any other finite Reynolds number. Depending on one's point of view, comparison with other theories appears to be reasonable. The Proudman–Pearson (1957) formula

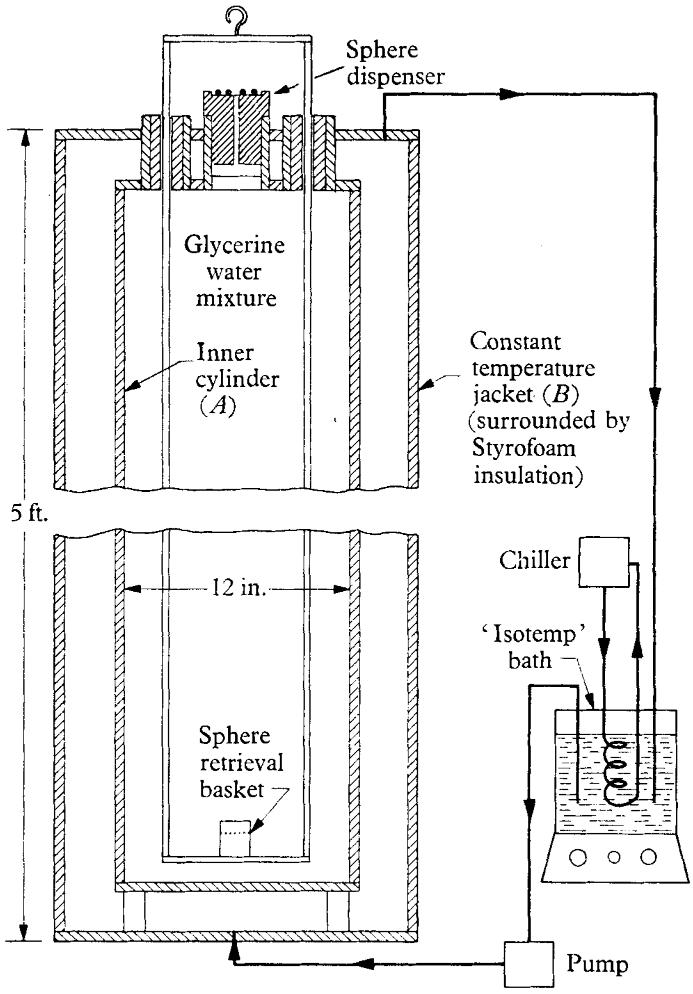

FIGURE 1. Apparatus.

is accurate to $1\frac{1}{2}$ % up to $R = 0{\cdot}65$, but represents the trend with R rather poorly; while the Goldstein (1929) formula is accurate to $1\frac{1}{2}$ % only up to $R = 0{\cdot}45$, but gives a particularly good representation of the behaviour of the drag as $R \to 0$.

Thus, it is the inescapable conclusion of this work that the formulae of Oseen and Goldstein represent the drag of a sphere most accurately for $R < 0{\cdot}45$, and that the addition of higher-order terms adds very little to the accuracy at low Reynolds numbers.

Thanks are due to E. J. Coury who constructed the major pieces of apparatus and assisted in the performance of the experiments, and to J. Krasinsky who measured the fluid density and viscosity with great accuracy.

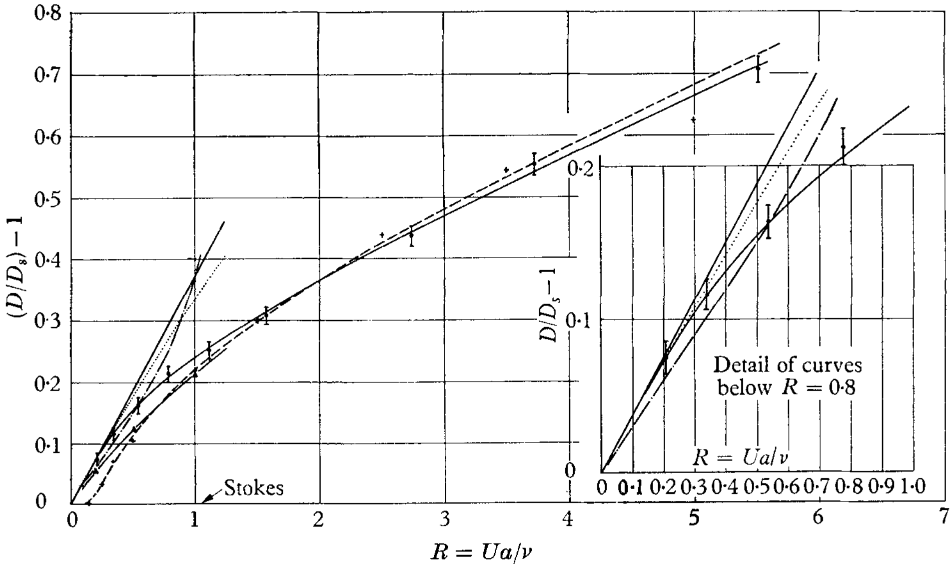

FIGURE 2. Sphere drag at low Reynolds numbers: $(D/D_s)-1$ against Ua/ν. —Ī—, present results; ——+——, Perry (1950); —▲—, Castleman (1925); ——, Oseen (1927), $(D/D_s)-1 = \frac{3}{8}R$; ...., Goldstein (1929), $(D/D_s)-1 = \frac{3}{8}R-\frac{19}{320}R^2+\frac{71}{2560}R^3$; ——-——, Proudman & Pearson (1957), $(D/D_s)-1 = \frac{3}{8}R+\frac{9}{40}R^2 \ln R$.

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